QIS4 Best Estimate Valuation Tool: User's Guide and Functionality Overview

The original document published in April has been revised to reflect the update to the Excel valuation tool.

Relevant changes are as follows:

- Sections 2.6 and 3.3: these have been amended to refer to the option (contained in the updated Excel tool) to specify an external tail factor, and to describe how this option can be used when the tool is applied
- Section 3.4: Paras. 58 to 61 have been added to give a more precise description of the estimation of the standard error for the tail, and to refer to the summary information (provided by the updated Excel tool) on the standard error of individual development factors relating to the data triangle.
- Sections 3.6 and 3.7: New paras. at the end of these sections have been introduced to describe how the calculations are carried out in case an external tail factor is specified.

Further, some small amendments have been made in section 3 to improve clarity and readability.

Contents

1   Introduction

1.1   Purpose of this Tool

1. Solvency II envisages a principle-based economic valuation of technical provisions, where the best estimate, defined as the expected present value of future cash flows, constitutes the main building block. For QIS4, CEIOPS has provided a Valuation Tool which is intended to assist participants in the valuation of the best estimate where this relates to claims provisions in the field of non-life insurance. This document describes how this tool can be used, and also provides details of the mathematics behind the various formulae.

2. The Valuation Tool is based on the Chain Ladder algorithm which is one of the base actuarial methodologies commonly used for determining best estimates of claims provisions. Rather than attempting to address the needs of actuarial experts in the field of claims reserving, it has been designed as a simple tool with a view to assist those insurers participating in the QIS which, up to now, have not used actuarial techniques or software as their primary means for setting provisions in non-life insurance.

3. To ensure that the tool can be easily applied, only the chain ladder algorithm – with a restricted number of further options – has been implemented. The tool derives estimates of the undiscounted as well as discounted best estimate of claims provisions on the basis of insurer-specific run-off triangles, and further provides the user with a number of additional information and functions which can be used to analyse and validate the results obtained:

    *   Calculation of the **chain-ladder factors** and of the consequent future payment flows (the prediction);

    *   Extension of a run-off trapezium into a run-off triangle by means of externally defined chain-ladder factors;

    *   Adjustment of a **tail function** estimating those payment flows which exceed the observed length of the run-off triangle;

    *   Calculation of the **prediction error** according to T. Mack for future payment flows;

    *   Calculation of the average **run-off time** and **duration** of payment flows; and

    *   Calculation of the **discounted provisions**, including the upwards- and downwards shocks provided by CEIOPS to asses the interest rate risk.

4. Due to its simplicity, the tool should only be used in cases where insurers have adequate historical claims data available which is sufficiently "smooth", i.e. the run-off patterns in the individual accident years should be comparable, without severe distortions due to e.g. extreme events, mergers or acquisitions or changes in claims settlement practices.

5.   Overall, it should be stressed that in the future Solvency II regime, any application of statistical claims reserving techniques would need to be supplemented by a range of actuarial "checks and balances" analysis, which would be outside the scope of the Valuation Tool or this guide. Whilst participants in the QIS4 are encouraged to take part on a "best efforts" basis, it seems unlikely that under the future Solvency II framework a simple "mechanical" application of a claims reserving algorithm would in itself be regarded as an appropriate valuation technique.

1.2   What this Tool does <u>not</u> intend

6.   CEIOPS is aware that a wide range of actuarial technique for claims reserving has been developed in the actuarial literature, and that the algorithms implemented in the tool represent only a small fraction of this range being subject to certain prerequisites with respect to data. However, this tool is not intended to compete with sophisticated or professional software in the field of actuarial claims reserving, and has deliberately been kept simple to achieve an easy applicability. It is in the responsibility of each participant to decide whether or not its use would lead to an appropriate valuation according to Solvency II principles.

7.   It has also not been CEIOPS' intention to analyse or discuss the merits or drawbacks of individual actuarial techniques within the context of this tool – there are many publications which describe the steps involved and the potential pitfalls.[1]

1.3   System requirement

8.   The valuation tool has been developed using Excel 2003. In case other versions of Excel (higher or lower, or with another language installed) are used, some menu items mentioned in this documentation or shown in screenshots may have changed.

1.4   Disclaimer

9.   This is a cost-free product provided by CEIOPS which is based on an Excel tool developed by the German insurance industry association (GDV) for the use in Solvency II quantitative impact studies. It has been validated by the competent staff members and bodies to the best of their knowledge and belief. Nevertheless, the tool may include technical or other mistakes, inaccuracies or typographical errors. Any feedback to this effect would be welcomed by CEIOPS. CEIOPS assumes no responsibility for errors or omissions in the tool, which is provided 'as is' without warranty of any kind, either express or implied.

---

[1] For example, the UK Actuarial Profession has set out a number of considerations that should apply when estimating general insurance provisions (available on their website www.actuaries.org.uk).

2 How to use the Valuation Tool

10. This section describes the functioning of the valuation tool. To get started, the user has to:

- Import the necessary **input data** into the spreadsheet (cf. 2.1);

- Determine (if applicable) any **pre-defined chain ladder factors** (cf. 2.2); and

- **Start the calculations** by applying the built-in macro (cf. 2.3).

The **results** of the calculations performed by the tool are described in sub-section 2.4. The section concludes by explaining how **subsequent applications** of the macro can be performed (cf. 2.5) and by describing **further options** provided by the tool (cf. 2.6).

2.1 Input of data triangles

11. The valuation tool uses the basic Chain Ladder algorithm[2] (based on paid claims) to derive a best estimate of the claims provision. As input data, this algorithm requires information on the (cumulative) paid claims in individual accident and development years.

12. As shown in Figure 1, these **cumulative payments** are entered in the worksheet *cumulative data* in the form of a run-off triangle or a trapezium (more accident years than development years). Trapeziums which comprise more development years than accident years cannot be processed. The bottom-left corner of the triangle has to be located in the cell *cumulative data*!B31, which is set off in red.

	A	B	C	D	E	F	G	H	I	J
16										
17										
18										
19										
20										
21										
22										
23		58.046	127.970	476.599	1.027.692	1.360.489	1.647.310	1.819.179	1.906.852	1.950.105
24		24.492	141.767	984.288	2.142.656	2.961.978	3.683.940	4.048.898	4.115.760	
25		32.848	274.682	1.522.637	3.203.427	4.445.927	5.158.781	5.342.585		
26		21.439	529.828	2.900.301	4.999.019	6.460.112	6.853.904			
27		40.397	763.394	2.920.745	4.989.572	5.648.563				
28		90.748	951.994	4.210.640	5.866.482					
29		62.096	868.480	1.954.797						
30		24.983	284.441							
31		13.121								

Figure 1: Run-off triangle in the input area

---

[2] The Chain Ladder algorithm is a statistical method of estimating outstanding claims, whereby the weighted average of past claim development is projected into the future. The projection is based on the ratios of cumulative past claims, usually paid or incurred, for successive years of development.

13. A run-off triangle from another Excel workbook may be entered by transferring it using the *copy and paste* function of Excel: having copied the triangle to be transferred, the user has to select in column B the cell in the row z ($z = 32 - \#\text{accident years}$). Then, via the menu item *Edit→Paste Special...*, in the window *Paste Special* (see Figure 2) the item *Values* is to be selected and the triangle is to be entered by means of the *OK* button.
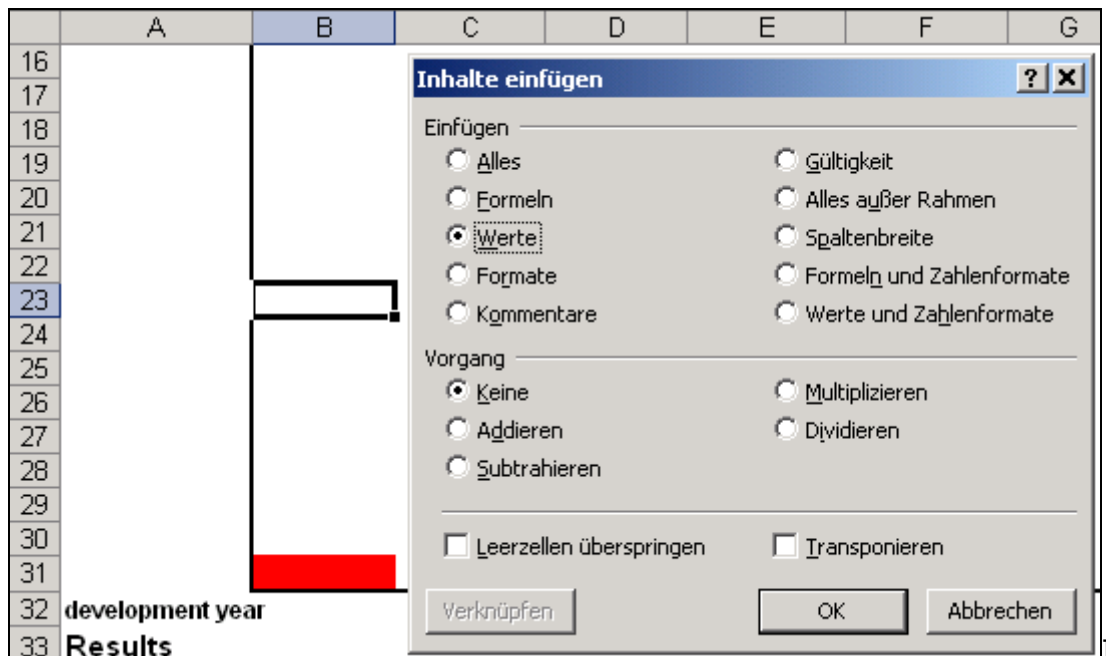

Figure 2: Inserting a copied triangle

14. It is necessary to enter at least 2 accident years and 2 development years. Moreover, no more than 30 accident years and development years may be entered.

> **Please note:**
> - It is important that the bottom left corner of the run-off triangle is located in the above-mentioned cell B31. If this is not the case, the "copy" command has to be reversed and the step has to be carried out once again.
> - This Excel tool requires <u>cumulative</u> (paid) claim amounts as input data. If incremental claims or incurred claims are entered, the tool will produce incorrect results.

2.2 Use of pre-defined development factors

15. The valuation tool allows the usage of pre-defined chain-ladder factors, e.g. from market sources. The tool uses these pre-defined factors to supplement the factors derived on basis of the insurer-specific input triangle. This may be helpful in cases where the available input triangle itself would be too short to determine appropriate factors for a sufficient number of

development years, even allowing for the tail factor completion provided by the tool (cf. section 3.3).

16. To use this option, the user should first select the line of business to which the triangle input data belongs. This is done by selecting in the combination field (cell *cumulative data*!D1) one of the possibilities (*Motor liability*, *accident*, *private liability*, *industry liability* or *Other lines of business*) (see Figure 3).
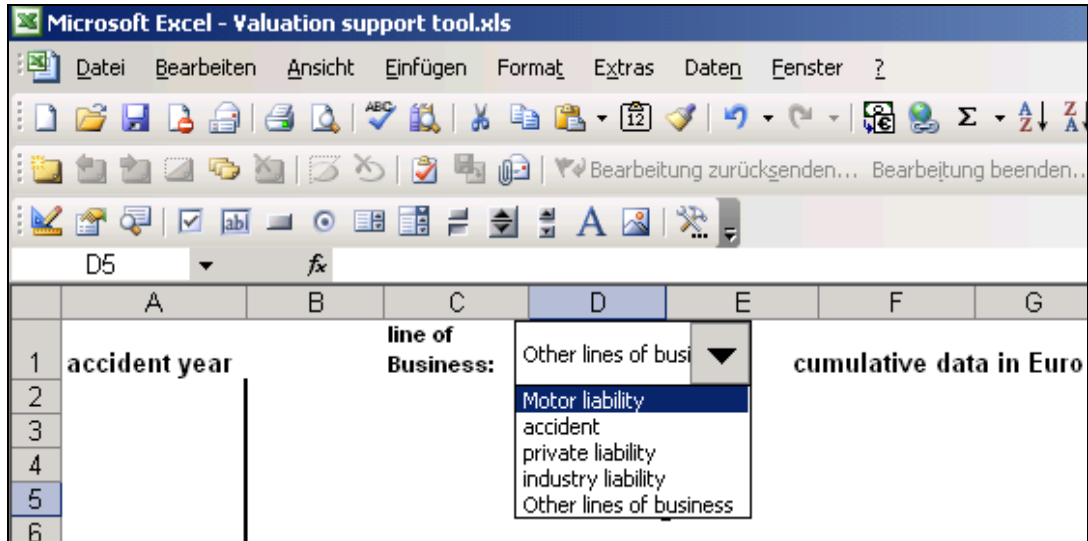

Figure 3: Selecting line of business

17. The pre-defined chain-ladder factors for the line of business in question need then be stored in the worksheet *parameters* (see figure 4). It is important that the factor list starts in the second row of the worksheet, and that there are no empty cells in the list.

	Dev. Year	Motor third party liability	Accident	Private liability	Industry liability	Other LOB
2	0					
3	1					
4	2					
5	3					
6	4					
7	5					
8	6					
9	7					
10	8					
11	9					
12	10					
13	11					
14	12					
15	13					

Figure 4: Worksheet for predefining chain-ladder factors

18. If a trapezium is entered, the number of accident years may not exceed the number of predefined chain-ladder-factors (plus one) because otherwise it cannot be completed into a run-off triangle.

> **Please note:**
>
> - The tool does not require the use of pre-defined factors. In case this option is not used, the selection of a line of business in the worksheet *cumulative data* does not affect the calculations.
>
> - When using pre-defined factors, the user should carefully consider whether the use of these factors would appropriately reflect the risk characteristics of the insurance portfolio to which they are applied.
>
> - The additional information on measurement error which the tool provides (see below) does not include any of the development years where pre-defined factors were used because the new "observations" exactly agree with the forecasts of the model.

2.3   Starting the calculations

19. Having provided the input data (and, if applicable, any pre-defined chain ladder factors) the macro performing the necessary calculations can be started. For this, the menu item *Extas → Macro → Macros…* may be selected, opening the macro dialogue box (also accessible via the shortcut *Alt+F8*). Here the macro *Evaluation* may be selected and started by activating the button *Run* (see Figure 5a).
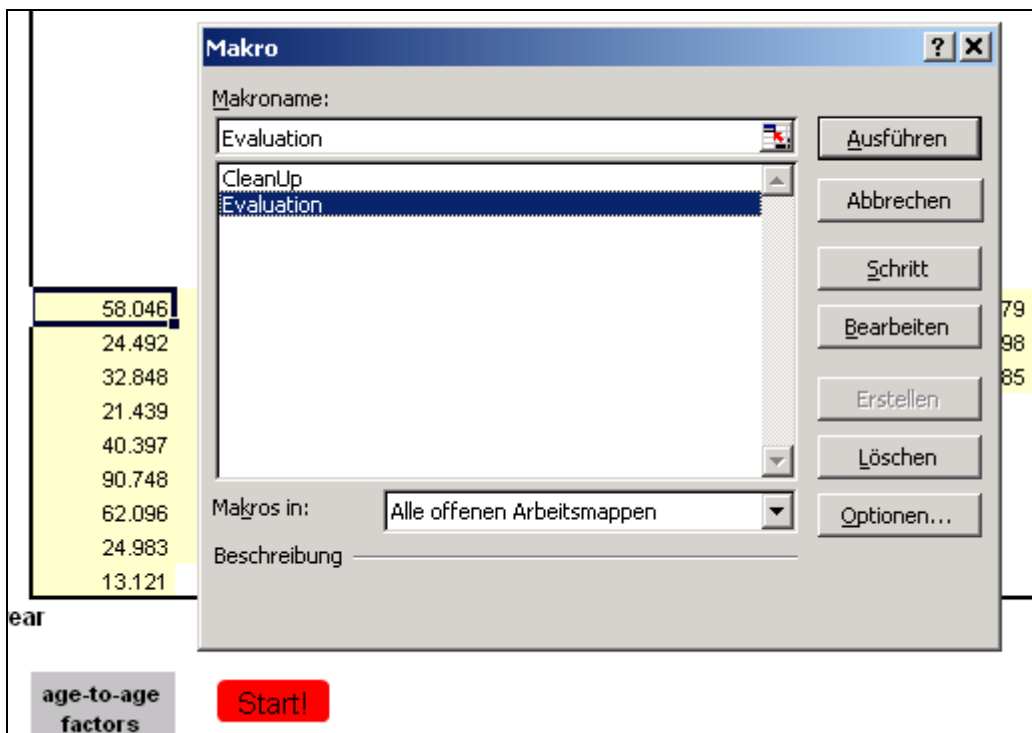

Figure 5a: Starting the macro via the menu

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 –
Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

8/26

20.　Alternatively, the macro may be started by a click with the left mouse button on the start button which is located in the cell *cumulative data*!C33 (see Figure 5b).

Figure 5b: Starting the macro by means of the start button

2.4　Results

21.　Once the built-in macro has been started, the calculations are performed automatically and all results are shown in the worksheets *results*, *cumulative data* and *future cashflows*.

22.　The main result of the calculations - the size of the **best estimate provision** – is summarised in the worksheet *results*. This comprises the undiscounted best estimate as well as discounted values based on the QIS4 term interest rate term structure for 2007 and the relevant upwards- and downwards shocks.[3] As additional information, the worksheet *results* also shows (see figure 5a):

- **Forecasts of the undiscounted future cashflows** for the next 10 calendar years;

- An estimate of the **modified duration** of the liability; and

- An estimate of the **prediction error** underlying the determination of the best estimate, following the model proposed by Thomas Mack (cf. sub-section 3.3 and 3.3, below), and an estimation of various **quantile levels** based on the standard error derived by the Mack model.

> **Please note:**
>
> The estimation of the prediction error and the quantile values provided here is not immediately relevant to any of the QIS4 calculations. A large

---

[3] Note that the term structure used for discounting (together with the up- and down-shocks) is read out by the macro from the worksheet *future cash-flows*!C2:AZ4 (cf. para. 73). The user may change these pre-settings in case a different year basis or a currency other than the Euro is used.

prediction error would generally indicate that further analysis is necessary, and that less reliance can be based on the best estimate derived by the tool.

In particular, the estimation of the various quantile levels is not intended to be used for the determination of the risk margin in technical provisions.
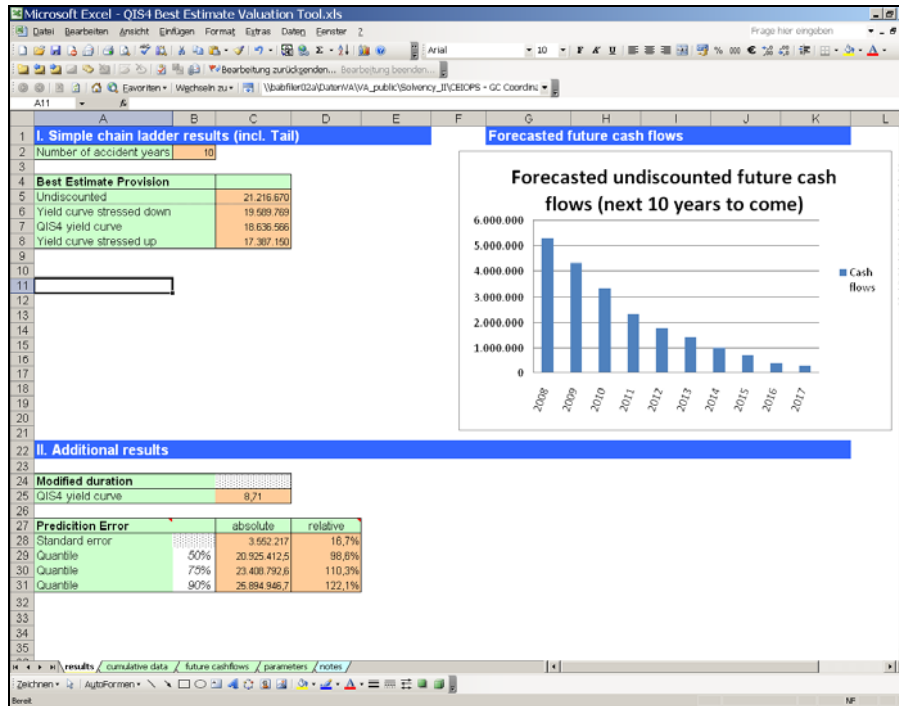

Figure 6a: Summary of results provided by the worksheet *results*

23. In the worksheet *cumulative data*, the following further details are shown concerning the estimated undiscounted future cash flows (see figure 6b):

- A "full" triangle or trapezium where the forecasted cumulative cash flows including a tail estimate (shown in blue) supplement the observed paid amounts;

- The chain ladder age-to-age factors in the individual development years, together with their coefficient of variation (according to Mack);

- The average run-off time of the provisions in individual accident years and information about the log-linear fitting of the tail function;

- Information on the required reserve and the prediction error (split between random error and estimation error) for the individual accident years, as well as for the overall results (both including and excluding the tail).

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 – Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

10/26

24. Further details concerning these results, including a description of the underlying mathematical formulae, are provided in section 3.

Figure 6b: The completed triangle shown in *cumulative data*

25. The worksheet *future cashflows* gives further information on (see figure 6c, below):

- The incremental future cash flows forecasted for the individual accident and development years;

- The undiscounted and discounted sum of outstanding cash flows in the individual accident years; and

- The absolute as well as modified duration of the provisions.

Figure 6c: Information on discounted values and duration in *future cashflows*

Further explanations concerning these results are given below (see subsection 3.7).

2.5 Subsequent applications

26. During the run of the macro, results are displayed also in the input area of the worksheet *cumulative data*. Thus, the triangle or trapezium is completed and extended by at least one column (accumulated total claims amount including tail estimate). In case the user wishes to run the macro on the same input data once again, for instance with changed options (see Section 2.2), **it is important that this additional column is deleted before the macro is restarted.** This can be done in two ways:

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 –
Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

11/26

- Run of the macro *CleanUp*: This macro deletes the contents of all cells on the worksheets *cumulative data and future cashflows* which accept changeable contents. Exceptions are options which have been entered by the user, hence the selection of the line of business and the contents of the cells J45 and S45 and Z45 of the worksheet *cumulative data*, highlighted in blue, and of the cells B1 and C2:AZ4 on the worksheet *future cashflows*. However, the user now has to enter the run-off triangle once again.

- Deletion of the additional columns: The run-off triangle or trapezium is trimmed to its original size. Additional columns are deleted manually. Please note that values below the diagonal are ignored when processing the input data, so that it is not necessary to restore the original form of the triangle or trapezium.

2.6 Further options

27. When determining the best estimate, the macro automatically adds a **tail factor** to allow for the development of claims beyond the development years corresponding to the claims triangle. This tail factor is obtained by fitting an exponential tail function on basis of a log-linear regression technique (cf. section 3.2). In this context, the following options are available:

- The user may pre-define the development year from which log-linear regression is to start if "other" is chosen as line of business (cf. section 3.2)

- The user may pre-define an external tail factor which is used instead of the estimated tail factor by means of the fitted tail function (cf. section 3.2)

- The user may pre-define an estimation error and a random error for the tail if external calculations are available for this (cf. section 3.4)

28. As a further option, the user may predefine the value "p" for which a p% quantile of the reserve is calculated (cf. section 3.5).

3 The mathematics behind the formulae

29. In the following, the mathematical formulae and algorithms underlying the calculations results shown in the worksheets *cumulative data and future cashflows* are described. More detailed descriptions of the mathematical framework, in particular with regard to the estimation of the prediction error contained in the calculations, can be found in [1] and [2].

3.1 Chain-ladder method

30. We consider cumulative data which are arranged in the form of a run-off triangle S_{ik} the rows and columns of which are numbered from $0,\dots,n$ respectively. The triangle is entered in the area *cumulative data!B2:AE31*. Using this run-off triangle and the formula

(1)
$$\hat{f}_k = \sum_{i=0}^{n-k-1} S_{i,k+1} \Big/ \sum_{i=0}^{n-k-1} S_{i,k} \ , 0 \le k \le n-1$$

or, equivalently,

$$\hat{f}_k = \sum_{i=0}^{n-k-1} \frac{S_{ik}}{\sum\limits_{i=0}^{n-k-1} S_{ik}} \cdot F_{ik} \ , \text{ with } F_{ik} := \frac{S_{i,k+1}}{S_{ik}}, \ 0 \le k \le n-1$$

it is now possible to calculate the n chain-ladder factors

$$\hat{f}_k, \ 0 \le k \le n-1,$$

which are shown in the area *cumulative data!B35:AD35* under the designation "best estimate" (BE). By means of these chain-ladder factors the run-off triangle is subsequently completed so as to estimate the ultimate loss and the reserves:

(2)
$$\hat{S}_{in} = S_{i,n-i} \cdot \hat{f}_{n-i} \cdot \dots \cdot \hat{f}_{n-1}$$

(3)
$$\hat{R}_i = \hat{S}_{in} - S_{i,n-i} = S_{i,n-i} \left(\hat{f}_{n-i} \cdot \dots \cdot \hat{f}_{n-1} - 1 \right), \text{ for i=1,....,n respectively.}$$

The reserves (3) are displayed row by row, starting in cell *cumulative data!B44*.

31. The chain-ladder factors are also used to estimate future payment flows $\hat{Z}$, which are shown in the worksheet *future cashflows* where they are used to determine discounted reserves:

(4)
$$\hat{S}_{i,k+1} = \hat{S}_{ik} \cdot \hat{f}_k, \quad \left(\text{where } S_{i,n-i} = \hat{S}_{i,n-i} \right)$$
$$\hat{Z}_{i,k+1} = \hat{S}_{i,k+1} - \hat{S}_{ik}, \quad for \ k = n-i, n-i+1, \dots, n-1.$$

3.2 Prediction error according to Mack

32. As has been laid out in section 2, the valuation tool uses the so-called Mack Model to derive – as an additional piece of information – an estimate

of the prediction error inherent in the determination of the best estimate. In the following, the mathematical formulae of the Mack Model are described in more detail.

33.   We note that the assessment of uncertainty within the reserving process is a topic which in recent years has received increased research and discussion in the actuarial profession, and there are currently a wide range of different stochastic methods in use for this purpose.[4]

34.   Whereas the Mack Model is one of those methods which are widely accepted by the profession, it uses a number of key assumptions on the underlying data, and may produce poor results if these assumptions are violated. Therefore, further actuarial judgement and analysis would generally be required to assess whether the prediction error derived by the Mack Model would indeed be an appropriate reflection of the uncertainty inherent in the best estimate calculation. For the purposes of the Valuation Tool, the prediction error should hence be seen as a tentative and indicative rather than as a definite assessment. In particular, a large prediction error would indicate that the best estimate results may contain a high degree of uncertainty.

35.   Let

$$mse\left(\hat{R}_i\right) \quad (i = 1,...,n)$$

denote the mean squared error, i.e. the prediction error of the reserve in the accident year i, while

$$se\left(\hat{R}_i\right) = \sqrt{mse\left(\hat{R}_i\right)} \quad (i = 1,...,n)$$

denotes the standard error, i.e. the square root of the prediction error in the accident year i. The standard error is important because it has the unit of a currency and hence the same unit as the cumulative payments of the run-off triangle. The prediction error for the estimates for the ultimate loss or for the reserve according to Mack is composed of two elements, namely the *random (or process) error* (RE) and the *estimation error* (EE).

36.   According to the Mack Model, the random error RE_i and the estimation error EE_i for an individual accident year i (where $1 \leq i \leq n$) may be determined as follows:

$$RE_i = \hat{S}_{in}^2 \sum_{k=n-i}^{n-1} \frac{\hat{\sigma}_k^2 / \hat{f}_k^2}{\hat{S}_{ik}} \ and \ EE_i = \hat{S}_{in}^2 \sum_{k=n-i}^{n-1} \frac{\hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{j=0}^{n-k-1} S_{jk}},$$

where

---

[4] A review of the practical and theoretical aspects of a few of the more common models for the assessment of uncertainty in setting claims provisions (including the Mack model) can e.g. be found in the 2007 General Insurance Convention paper "Best Estimates and Reserving Uncertainty" (available under www.actuaries.org.uk).

$$\hat{\sigma}_k^2 = \frac{1}{n-k-1} \sum_{i=0}^{n-k-1} S_{ik} \left(\frac{S_{i,k+1}}{S_{ik}} - \hat{f}_k \right)^2 \quad for \ n-i \le k \le n-1$$

and

$$\hat{\sigma}_{n-1}^2 = \min(\hat{\sigma}_{n-2}^4 / \hat{\sigma}_{n-3}^2, \min(\hat{\sigma}_{n-3}^2, \hat{\sigma}_{n-2}^2))$$

37. It then follows that the overall prediction error for an individual accident year i (as the sum of random error and estimation error) is given by

(5)
$$mse\left(\hat{R}_i\right) = \hat{S}_{in}^2 \sum_{k=n-i}^{n-1} \frac{\hat{\sigma}_k^2}{\hat{f}_k^2} \left(\frac{1}{\hat{S}_{ik}} + \frac{1}{\sum_{j=0}^{n-k-1} S_{jk}} \right).$$

38. The root of the prediction error is displayed as a column starting in cell *cumulative data*!C44. The adjacent column from B44 shows the respective estimated reserve. The following column from D44 shows the ratio between the standard error and the reserve. The columns below E44:H44 show the roots of the random error and of the estimation error as well as the ratios between the random error and the prediction error and between the estimation error and the prediction error.

39. The parameters

$$\hat{\sigma}_k \quad (k = 0,...,n-1)$$

are shown in the area *cumulative data*!B38:AD38. The area *cumulative data*!B41:AD41 shows the estimator for the standard deviation of the chain-ladder factors:

$$\sqrt{Var\left(\hat{f}_k\right)} = \sqrt{\hat{\sigma}_k^2 / \sum_{j=0}^{n-k-1} S_{jk}}, \text{ with } k = 0,...,n-1.$$

40. The coefficient of variation *Vko*, which is displayed in the area *cumulative data*!B36:AD36, is calculated as follows:

$$Vko_k = \sqrt{Var\left(\hat{f}_k\right)} / \hat{f}_k, \quad \text{with} \quad k = 0,...,n-1.$$

41. On basis of the prediction error for individual accident years i the prediction error of the overall reserve may be derived as follows:

(6)
$$mse(\hat{R}) = \sum_{i=1}^{n} \left\{ \left(se\left(\hat{R}_i\right)\right)^2 + \hat{S}_{in} \left(\sum_{j=i+1}^{n} \hat{S}_{jn} \right) \sum_{k=n-i}^{n-1} \frac{2\hat{\sigma}_k^2 / \hat{f}_k^2}{\sum_{m=0}^{n-k-1} S_{mk}} \right\}.$$

42. The standard error for the overall reserve is then given by the square root of the prediction error:

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 – Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

15/26

$$se(\hat{R}) = \sqrt{mse(\hat{R})}$$

This quantity (which has the same unit as the original data) is shown under the heading "standard error of the overall reserve" in the cell *cumulative data*!K44. This is followed by the ratio between the standard error and the overall reserve in the cell L44 (the overall reserve itself is shown in the cell J44).

43.   In case a run-off trapezium with $n+1$ accident years and $k_0 < n+1$ development years is evaluated, the following applies to the parameters introduced above (cf. para. 18):

$$\hat{\sigma}_{k_0} = \ldots = \hat{\sigma}_{n-1} = 0,$$

$$Var\left(\hat{f}_{k_0}\right) = \ldots = Var\left(\hat{f}_{n-1}\right) = 0,$$

$$Vko_{k_0} = \ldots = Vko_{n-1} = 0,$$

$$mse\left(\hat{R}_1\right) = \ldots = mse\left(\hat{R}_{k_0}\right) = 0,$$

$$RE_1 = \ldots = RE_{k_0} = EE_1 = \ldots = EE_{k_0} = 0.$$

3.3 Tail function

44.   All formulae shown so far refer to a determination of the best estimate without considering the "tail" of the claims development beyond the last development period covered by the input data. However, in practice, an allowance for such tail development of claims costs often needs to be made in order to derive appropriate ultimate loss estimates. To achieve this, the valuation tool determines a tail factor either internally (via fitting a tail function using a log-linear regression) or externally (by using an exogenous tail factor specified by the user), as is described in this section. The macro then also determines random and estimation errors for the best estimate calculations including the tail factor, as explained in section 3.4.

45.   For the internal estimation of the tail of the claims development, an exponential development function

(7) $f(k) = 1 + a \cdot \exp(b \cdot k),\ b < 0.$

is fitted to the chain-ladder factors.

46.   If necessary, the chain-ladder factors of the run-off triangle are supplemented by adding the predefined chain-ladder factors if development years are missing in the input data. Thus, the length of the run-off triangle changes from *n* to

$n' := \max(n, w),$

where w denotes the sum of the number of original chain-ladder-factors and the number of additional predefined chain-ladder-factors (if any).

47.   The coefficients a, b of the tail function are obtained by means of log-linear regression: For this regression use is made of the numbers of the

development years used as independent variable x and of the logarithm of the chain-ladder factors minus one as dependent variable $y = bx + a$[5]:

(8)
$$x = \left(r, r+1, \ldots, n'-1 \right)$$
$$y = \left(\ln\left(\hat{f}_r - 1 \right), \ln\left(\hat{f}_{r+1} - 1 \right), \ldots, \ln\left(\hat{f}_{n'-1} - 1 \right) \right)$$

48.   In this case the index r denotes the development year from which the regression starts depending on the line of business. The following table lists the development years used according to the selected line of business.

Line of business	From development year r
Motor liability, accident, liability (private and industry)	5
Other	Choice of the user

49.   If "other" has been chosen as line of business, the user selects the development year from which the regression starts. For this purpose the user enters the selected value in the cell *cumulative data*!Z45 (see Figure 7). A change of this value is not applied until the macro is started once again.

Figure 7: Choice of the development year and predefining a tail factor

Note that the algorithm requires at least two development years to perform the tail regression. Therefore, the value of r should not exceed $n'-2$, where n' denotes the length of the run-off triangle.

50.   The function $f(k)$ now provides development factors for $k \geq n'$. If these factors are multiplied by each other, an estimate for the tail factor f_{ult} is obtained by which the ultimate loss exceeds the accumulated total claims amount after development year n'.

(9) $$\hat{f}_{ult} = \prod_{k=n'}^{\infty} f(k).$$

---

51. The coefficients of the tail function, the tail factor and the run-off time of the tail are displayed in the cells *cumulative data*!W44:Z44. The infinite product converges and practice shows that generally a sufficiently good approximation to the limit is reached after 50 years $(k = 49)$.

52. If the user already has an estimate of a tail factor for the given run-off triangle, he may use this factor directly instead of the estimation by means of the tail function. To do so the user has to input the value into the cell Y44 before the calculation is started (see Figure 7, above). In this case the fitting of the tail function is suppressed and all the calculations that depend on the tail factor are carried out by using the input value.

Please note:

The use of a tail function as in equation (7) has the advantage to provide development factors for each of the development years beyond the observed data triangle payments. This allows an allocation of future payments in the "tail" to individual future development periods. In contrast to this, in case a pre-set tail factor is used (and hence the fitting of a tail function is suppressed), it has to be assumed that each payment in the "tail" is allocated to a single development period (following the last observable development year).

Whereas this difference in the usage of a tail function vs. an external tail factor does not effect the calculation of the undiscounted best estimate, it will influence the determination of the run-off time as well as the discounted reserves. Therefore, where an externally derived tail factor is used, the user should consider whether these values are appropriately derived.

3.4 Prediction error according to Mack including the tail

53. In this section, unless otherwise stated, the following applies: $i = 0, \ldots, n$ and $k = 0, \ldots, n-1$. The mean squared error of the estimated ultimate losses $\hat{S}_{in}$ calculated according to Mack is identical with the mean squared error of the reserves $\hat{R}_i$:

$$(10) \qquad mse\left(\hat{R}_i\right) = mse\left(\hat{S}_{in}\right) .$$

54. Using this equation, and observing that

$$(11) \qquad mse\left(F_{ik}\right) = \left(se\left(F_{ik}\right)\right)^2 = \frac{\hat{\sigma}_k^2}{S_{ik}} \quad \text{and} \quad mse\left(\hat{f}_k\right) = \left(se\left(\hat{f}_k\right)\right)^2 = \frac{\hat{\sigma}_k^2}{\sum_{j=0}^{n-k-1} S_{jk}} ,$$

we find that formula (5) can be expressed in an equivalent way as follows:

$$(12) \qquad mse\left(\hat{S}_{in}\right) = \hat{S}_{in}^2 \sum_{k=n-i}^{n-1} \left(mse\left(F_{ik}\right) + mse\left(\hat{f}_k\right)\right) \Big/ \hat{f}_k^2 .$$

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 –
Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

18/26

55. The two equations contained in (11) may be interpreted as follows: The parameter $se(F_{ik})$ estimates the extent to which the individual settlement factors $\dfrac{S_{i,k+1}}{S_{ik}}$ deviate on average from the chain-ladder factor f_k while $se(\hat{f}_k)$ describes the extent to which the estimated chain-ladder factor $\hat{f}_k$ deviates from the actual chain-ladder factor f_k.

56. From formula (12) a recursive definition of the standard errors of the estimated accumulated total claims amounts may be derived:

(13) $$mse\left(\hat{S}_{i,k+1}\right) = \hat{S}_{i,k}^2\left(mse\left(F_{ik}\right) + mse\left(\hat{f}_k\right)\right) + mse\left(\hat{S}_{ik}\right)\hat{f}_k^2 \ .$$

The initial value for the recursion for each row is the diagonal element for which $mse\left(\hat{S}_{i,n-i}\right) = 0$ is assumed. By means of the recursive definition it is possible to calculate the standard error for the ultimate loss including the tail factor as follows:

$$mse\left(\hat{S}_{i,ult}\right) = \hat{S}_{in}^2\left(mse\left(F_{i,ult}\right) + mse\left(\hat{f}_{ult}\right)\right) + mse\left(\hat{S}_{in}\right)\hat{f}_{ult}^2 \ .$$

57. To apply this formula the following three parameters have to be estimated:

$$\hat{f}_{ult}, \quad se\left(\hat{f}_{ult}\right), \quad se\left(F_{i,ult}\right) \ .$$

These are estimates for the tail factor, the standard error of the tail factor (estimation error) and the standard errors of the individual development factors for the tail (random error). The estimate for the tail factor is calculated by the macro as explained under Section 3.2. The other parameters have to be estimated by the user on the basis of the tail factor.

58. Concerning the standard error of the individual development factors for the tail, it follows from the assumptions of the Mack Model that

$$se\left(F_{i,ult}\right)\sqrt{S_{i,ult}} = \hat{\sigma}_{ult} = se\left(F_{j,ult}\right)\sqrt{S_{j,ult}}$$

for each pair i,j of accident years. Hence in order to estimate the standard errors $se\left(F_{i,ult}\right)$ for each of the accident years $i=0,\ldots,n$, it suffices to estimate $se\left(F_{i_0,ult}\right)$ for an individual index i_0; the standard errors corresponding to other accident years may then immediately be derived from the formula above. This approach is used by the macro, where for reasons of simplification the third accident year is used as an "anchor point" to derive the standard errors for the other accident years (i.e. $i_0 = 2$ is chosen).

59. Hence in order to determine the standard error for the ultimate loss including the tail factor it suffices overall to specify two parameters, i.e. the

standard error of the tail factor and the standard error of the tail development factor corresponding to accident year $i_0 = 2$.

60. The macro uses a simple rule of thumb to estimate the two parameters as follows:

$$se\left(F_{i_0,ult}\right) = se\left(\hat{f}_{ult}\right) = \left(\hat{f}_{ult} - 1\right)\Big/2 .$$

61. If this rule of thumb should not be used, the user has to enter values in the cells L1 and N1 in the worksheet *cumulative data*:

I	J	K	L	M	N	C
for tail estimates:		s.e.(f_ult)	2,0%	s.e.(F₁,ult)	3,0%	

Figure 8: Entering the estimators

Changes in the entries of the two cells are not taken into account until a new run of the macro To assist users who wish to specify these two parameters themselves, the valuation tool provides an overview of the parameters

$$se\left(F_{i,k}\right) \;\; (for\; i = 0,...,n) \;\; \text{and} \;\; se\left(\hat{f}_k\right)$$

for each of the development years *k=0,…,n-1* in the worksheet *s.e. analysis*.

62. The results described in this section are shown in the columns *cumulative data!N:U* from the row 44. Details on the subject of prediction error including tail may be read in [2].

Please note:

If the run-off triangle is continued with pre-defined chain-ladder-factors, no volatility parameters $\hat{\sigma}$ for development years obtained in this way are calculated, since these development years add nothing to the prediction error including the tail.

3.5 p% quantile

63. For the overall reserve without and with tail a p% quantile q is calculated based on a log-normal distribution using the parameters μ, σ^2:

$$q = \exp\left(\mu + \sigma \cdot \Phi^{-1}(p)\right), \;\; 0 < p < 1 .$$

64. The parameters μ, σ^2 are derived, for instance, according to the method of moments from the expected value and the standard deviation of the log-normal distribution. For both parameters the estimators from the chain-

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 –
Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

20/26

ladder method are used: $\sum_{i=0}^{n} \hat{R}_i = R$ for the expected value and the prediction error according to Mack $mse(R)$ as variance according to the formulae:

$$\sigma^2 = \ln\left(1 + mse(R)/R^2\right)$$
$$\mu = \ln(R) - \sigma^2/2$$

65. The percentage p may be freely determined by the user both for the overall reserve without tail and for the overall reserve with tail independently of each other by entering the value $p \cdot 100$ on the worksheet *cumulative data* in the cells J45 (without tail) or S45 (with tail) (75 instead of 0.75; see Figure 9). Changes in the value entered in these cells are not taken into account until a new run of the macro.

Figure 9: Entering the p% quantile

3.6 Run-off time

66. The run-off time of payment flows for observed years $k = 0, \dots, n-i$ and forecasted years $k = n-i+1, \dots, n'$ is calculated using the annual median points $t_k = k + \dfrac{1}{2}$ separately for each accident year:

$$d_i := \sum_{k=0}^{n'} \frac{Z_{ik}}{\sum_{p=0}^{n'} Z_{ip}} t_k = \frac{1}{S_{in'}} \sum_{k=0}^{n'} Z_{ik} t_k \ , \ i = 0, \dots, n \ .$$

67. To calculate the run-off time for subsequent years greater than n' the tail function for estimating the future accumulated total claims amounts is used $\left(m := 49 - n'\right)$:

$$D_i = \sum_{k=1}^{m} \frac{Z_{i,n'+k}}{\sum_{p=1}^{m} Z_{i,n'+p}} t_{n'+k} = \frac{1}{S_{i,n'+m} - S_{in'}} \sum_{k=1}^{m} Z_{i,n'+k} t_{n'+k}, \ \ i = 0, \dots, n \ .$$

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 –
Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

21/26

68. From the two parameters d_i and D_i the averaged overall run-off time is calculated as follows:

$$G_i = \frac{S_{in'} \cdot d_i + \left(S_{i,n'+m} - S_{in'}\right) \cdot D_i}{S_{i,n'+m}}, \quad i = 0,\ldots,n.$$

The overall run-off time is displayed depending on the accident year in one column from the cell *cumulative data*!Q44.

69. In case an external tail factor is specified by the user, a simplified calculation of the overall run-off time is performed where the calculation of the parameter d_i includes the additional (single) cash flow corresponding to the tail factor, and where hence the calculation of parameter D_i becomes obsolete.

3.7 Discounting

70. For a given term structure z_k, $k = 0,1,2,\ldots$ for the balance sheet year J the present value of the future payments Z^d is calculated as follows:

$$Z_{ik}^d := Z_{ik} \frac{1}{\left(1 + z_{k-n+i-1}\right)^{k-n+i-0,5}}, \quad i = 0,\ldots,n, \; k = n-i+1,\ldots\ldots,\infty \, (49).$$

The interest rate $z_k, k > 0,$ applies to the period $\left(J, J+k\right]$ whereas the interest rate z_0 applies to a half year's period.

71. In the special case of a constant interest rate, the present value of a future payment flow in the accident year i ($Z_{i,k-n+1}, Z_{i,k-n+2}, \ldots$) is calculated analogically by means of the formula

$$B_i(z) := \sum_{k=n-i+1}^{49} \frac{1}{\left(1+z\right)^{k-n+i-0,5}} \cdot Z_{ik} \; \text{with} \; i = 0,\ldots,n.$$

72. The present value calculated by means of this formula is an approximative solution because the summation is only made until the development year 49 (with the same justification as in section 3.3).

73. The term structure used for these and the following calculations and its two variants are read out by the macro from the worksheet *future cash-flows*!C2:AZ4 (see Figure 10).

	A	B	C	D	E
1	constant average interest rate	3,000%			
2		Downward shock	2,301%	2,399%	2,526%
3	Term structure 2007	Standard	4,696%	4,526%	4,510%
4		Upward shock	9,110%	8,011%	7,622%
5	Future cashflows and discounted present value				
6					
7					

Figure 10: Changes in the term structures

74. The second row of this worksheet shows the values for the variant of interest rate reduction, the third row the standard term structure and the fourth row the variant for interest rate increase. The user may edit the individual values. Any changes are taken into account when the macro is rerun.

75. In the area *future cashflows*!B6:AJ35 the payments to be made in the future are displayed. The column which is underwritten with tail payments is followed by a column showing the total amount still to be paid for each accident year. This is followed by three columns which contain the present values of this total amount discounted with the three variants of term structures for each accident year. The first of these three columns (*disc.(downw.)*) refers to the scenario of an interest rate reduction, the second (*disc.*) to standard interest rates and the third (*disc.(upw.)*) to a rise in interest rates. Below these three columns the sums of the individual columns are shown, i.e. the present values of the entire portfolio discounted in three different ways and their difference in terms of percentage with respect to the undiscounted reserves including the tail (cell T44 on the worksheet *cumulative data*).

> **Please note:**
>
> The output in this worksheet shifts depending on the size of the evaluated run-off triangle.

76. To the right of the three columns with the interest rate scenarios, another six columns of data are shown. These contain the parameters which are dependent on the accident year, i.e. the absolute duration *(abs. dur.)* and the modified duration *(mod. dur.)* as well as an average modified total duration *(total)* both for a non-constant term structure and for a fixed interest rate.

77. To explain these parameters, let $B_i(z)$ denote the present value function which determines the present value of a payment flow at a fixed interest rate with an observation period of 50 years (development years 0 till 49).

The change of this function in the case of a change in the interest rate is described by a parameter which is referred to as *absolute duration* D_i^{abs} :

$$D_i^{abs}(z) := -\frac{dB_i(z)}{dz} = \sum_{k=n-i+1}^{49} (k-n+i-0,5) \cdot Z_{ik} \cdot (1+z)^{-(k-n+i-0,5)-1} > 0 \text{ with } i = 0,\ldots,n.$$

78. To interpret this parameter the Taylor expansion of the present value function is considered:

$$B_i(z+h) - B_i(z) = \frac{dB_i(z)}{dz} \cdot h + \underbrace{rest(h)}_{\to 0 \text{ for } h \to 0} \approx -D_i^{abs} \cdot h.$$

The above equation means that the present value B_i is reduced by $D_i^{abs}/100$ euro if the interest rate rises by 1 percentage point. Vice versa, the present value increases by $D_i^{abs}/100$ euro if the interest rate decreases by 1 percentage point.

79. If the absolute duration is divided by the present value $B_t(z)$, the modified duration D_i^{mod} is derived:

$$D_i^{mod}(z) := \frac{D_i^{abs}(z)}{B_i(z)} = \frac{\displaystyle\sum_{k=n-i+1}^{49} (k-n+i-0,5) Z_{ik} (1+z)^{-(k-n+i-0,5)-1}}{\displaystyle\sum_{k=n-i+1}^{49} Z_{ik} (1+z)^{-(k-n+i-0,5)}} \text{ with } i = 0,\ldots,n.$$

This parameter may be interpreted as follows: The present value is reduced by $(D_i^{mod} \cdot h)\%$ if the interest rate rises by h percentage points. Likewise, the present value increases by $(D_i^{mod} \cdot h)\%$ if the interest rate drops by h percentage points.

80. In structural terms, the modified duration may be compared to the average run-off time (cf. section 3.6): D_i^{mod} corresponds to an average run-off time exclusively weighted by the future discounted payment flows.

> **Example:**
>
> Suppose that $D_i^{mod} = 4$. Then in the case of an interest rate increase of 0.5 percentage points (for instance, from 3.5% to 4.0%) the present value is reduced by $(D_i^{mod} \cdot 0,5)\% = 2,0\%$.

81. In calculating this parameter depending on the accident year an interest rate should be used which matches the average term. The guideline for the average term is the overall run-off time according to section 3.6. To obtain parameters which are independent of the accident year, the follow-

ing procedure would be appropriate: For a fixed interest rate the modified durations for each accident year, weighted by the sums of all future payments in an accident year, may be averaged. $\bar{D}^{mod}$ is derived. By analogy, the formula for the modified duration when using a non-constant term structure is derived:

$$D_i^{mod}\left(z_1,\dots,z_t\right) = \frac{\sum_{k=n-i+1}^{49}\left(k-n+i-0,5\right)Z_{ik}\left(1+z_{k-n+i-1}\right)^{-\left(k-n+i-0,5\right)-1}}{\sum_{k=n-i+1}^{49}Z_{ik}\left(1+z_{k-n+i-1}\right)^{-\left(k-n+i-0,5\right)}}\,.$$

82.   From this, in turn, a modified duration which is independent of the accident year may be calculated in the form just described. The interest rates needed for calculating the parameters depending on a non-constant term structure are read out of the area *future cashflows*!C3:AZ3. If the durations are to be calculated for a fixed interest rate, this interest rate has to be entered in the cell *future cashflows*!B1 (see Figure 11). **Note**: If the user does not enter anything here, this is interpreted as a 0% interest rate. The non-binding standard preset here is 3%.

83.   In case an external tail factor is specified by the user, it is assumed that all payments in the "tail" of the development of future cash flows occur in the first development year for which no observable or pre-defined chain ladder factors are available, i.e. it is assumed that

$$Z_{ik} = 0$$

for all k > n'+1, where n' denotes the length of the data triangle including any additionally pre-specified chain ladder factors (cf. section 3.3).

CEIOPS e.V. – Westhafenplatz 1 - 60327 Frankfurt – Germany – Tel. + 49 69-951119-20 –
Fax. + 49 69-951119-19 email: secretariat@ceiops.eu; Website: www.ceiops.eu

25/26

4   Bibliography

[1] Thomas Mack, "Distribution-Free Calculation Of The Standard Error Of Chain Ladder Reserve Estimates", Astin Bulletin, Vol. 23, No.2, 1993

[2] Thomas Mack, "The Standard Error Of Chain Ladder Reserve Estimates : Recursive Calculation And Inclusion Of A Tail Factor", Astin Bulletin, Vol. 29, No.2, 1999